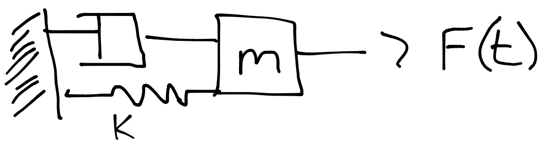
Vibrations Continued

1 degree of freedom → damped forced vibrations



$F(t) = F_0 \sin \omega t$

Various ways to shake something
- applying force
- shaking base

$\Big\}\ F(t)$

main equation: $m\ddot{x} + c\dot{x} + Kx = F_0(t)$

* divide through by m → $\ddot{x} + \frac{c}{m}\dot{x} + \frac{K}{m}x = \frac{F_0}{m}\sin\omega t$

* new variables → $\ddot{x} + 2\zeta\omega_n\dot{x} + \omega_n^2 x = F_0 \sin\omega t$

$\omega_n = \sqrt{K/m}$ = natural frequency

ζ = damping ratio = $\frac{c}{2\sqrt{Km}}$

$2\zeta\omega_n = \frac{c}{m}$

Solution of $x(t)$: $x(t) = x_h + x_p$

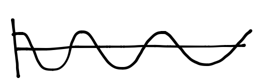
x_h: homogeneous solution, transient response

x_p: particular solution, steady-state response

Homogeneous (transient) solution

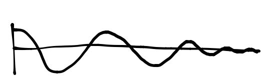
Set $F = 0$

- whole set of solutions



$c=0,\ \zeta=0$

c = big, ζ = big → overdamped



c = small → underdamped

c = biggish → overdamped

c = critcal

Critical: $\sqrt{c^2 - 4km} = 0 = 2\sqrt{km}$

X_h = linear combination of solutions

each: 1.) $\mathrm{Re}\, e^{\lambda t}$ λ: roots of → $\lambda = \dfrac{-2\zeta\omega_n \pm \sqrt{4\zeta^2\omega_n^2 - 4\omega_n^2}}{2}$

$= -\omega_n \zeta \pm \sqrt{\zeta^2 - 1}\, \omega_n$

$\lambda = \omega_n \left[-\zeta \pm \sqrt{\zeta^2 - 1} \right]$

Steady State Solution: X_p

Very slow Force: a sequence of static forcing

- force and position have the same phase

at very high freauency forces, the spring and dashpot don't do anything

resonance: $\omega = \omega_n$

Back to X_p:

$X_p = A\cos\omega t + B\cos\omega t = C\sin(\omega t - \delta)$, $C = \sqrt{A^2 + B^2}$

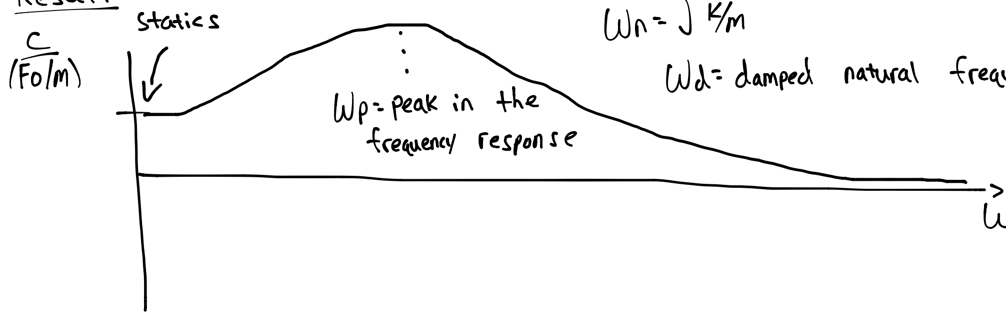
$\underbrace{A = B = C = \delta}_{= \text{mess}}$

(see Tongue 2.6.14)

(RP 10.33)

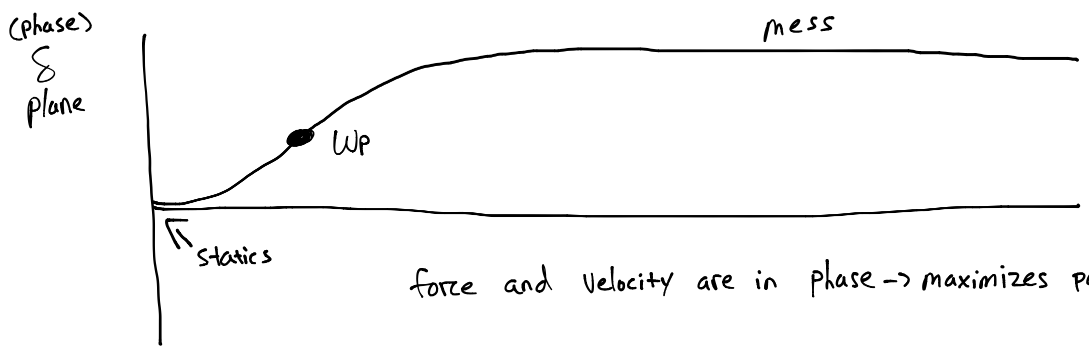
(Taylor 5.68)

Result:



$\omega_n = \sqrt{k/m}$

ω_d = damped natural frequency



force and velocity are in phase -> maximizes power

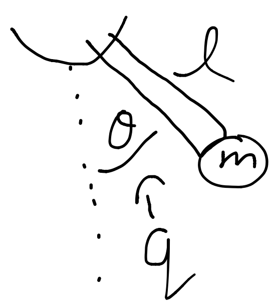
λ = 1. real roots
2. complex roots

Picking up on Lagrange Equations

For some class of problems, L.E's replace Newton's Laws

1 DoF systems (see previous lecture on LaGrange

Example: pendulum



$E_p = -mgl\cos\theta =$ "V"

$E_K = \frac{1}{2}m(l\dot{\theta})^2 =$ "T"

L.E's: $\frac{\partial \mathcal{L}}{\partial q} - \frac{d}{dt}\frac{\partial \mathcal{L}}{\partial \dot{q}} = 0$

$\mathcal{L} = E_K - E_P = \frac{1}{2}ml^2\dot{\theta}^2 + mgl\cos\theta$

L.E.'s $= -mg\ell\sin\theta - \frac{d}{dt} m\ell^2\dot{\theta} = 0$

$= -mg\ell\sin\theta - m\ell^2\ddot{\theta} = 0$

$\ddot{\theta} + \frac{g}{\ell}\sin\theta = 0$ $\rightarrow$ The pendulum equation of motion

has equilibrium solution at $\theta = 0$

- near equilibrium $\rightarrow$ $\theta << 1$
- gives linearized equation: $\ddot{\theta} + \frac{g}{\ell}\theta = 0$